

Algebraic Fractions

Patrons are reminded that if something looks like it factorises then factorise it. And don't make the donkey-like mistake

$$\frac{2+x}{3+x} = \frac{2}{3} \quad \text{or} \quad \frac{2+x}{3+x} = \frac{3}{4}.$$

You can only cancel when there is a multiplicative relationship; for example

$$\frac{2x+2}{x^2+x} = \frac{2(x+1)}{x(x+1)} = \frac{2}{x}.$$

1. Combine the following algebraic fractions, fully simplifying your answer.

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|---|------------------------------------|---|-------------------------------------|
| (a) $\frac{2}{x+4} + \frac{3}{x+1}$ | $\frac{5x+14}{(x+1)(x+4)}$ | (l) $\frac{5}{x^2+3x} + \frac{2}{x+3}$ | $\frac{2x+5}{x(x+3)}$ |
| (b) $\frac{3}{x-2} + \frac{5}{x+4}$ | $\frac{8x+2}{(x+4)(x-2)}$ | (m) $\frac{x}{x^2-x-6} - \frac{3}{2x+4}$ | $\frac{9-x}{2(x-3)(x+2)}$ |
| (c) $\frac{3}{x-1} - \frac{5}{x+2}$ | $\frac{11-2x}{(x-1)(x+2)}$ | (n) $\frac{x+1}{x^2-4x+3} - \frac{x-3}{x^2-1}$ | $\frac{8}{(x+1)(x-3)}$ |
| (d) $\frac{1}{x-5} - \frac{3}{x-7}$ | $\frac{8-2x}{(x-5)(x-7)}$ | (o) $\frac{2}{x} - \frac{3}{x^2-x} - \frac{4}{x-1}$ | $-\frac{2x+5}{x(x-1)}$ |
| (e) $\frac{3}{2x+4} - \frac{1}{3x+6}$ | $\frac{7}{6(x+2)}$ | (p) $\frac{1}{x-1} - \frac{1}{x^2+x-2} + \frac{4}{x+2}$ | $\frac{5x-3}{(x-1)(x+2)}$ |
| (f) $\frac{a}{x+b} + \frac{a}{x+c}$ | $\frac{2ax+ab+ac}{(x+b)(x+c)}$ | (q) $\frac{2}{x^2-x} - \frac{1}{x^2-1}$ | □ |
| (g) $2 + \frac{3}{x-5}$ | $\frac{2x-7}{x-5}$ | (r) $\frac{3}{2x-3} + \frac{1}{x-4} - \frac{5x}{2x^2-11x+12}$ | $-\frac{15}{(2x-3)(x-4)}$ |
| (h) $\frac{5}{2-x} + 3 + \frac{1}{x+1}$ | $\frac{3x^2-7x-13}{(x+1)(x-2)}$ | (s) $\frac{a}{x+1} - \frac{b}{x^2+2x+1} - \frac{c}{(x+1)^3}$ | $\frac{ax^2+2ax-bx+a-b-c}{(x+1)^3}$ |
| (i) $\frac{a}{x+k} + \frac{b}{x+2k}$ | $\frac{ax+bx+2ak+bk}{(x+k)(x+2k)}$ | | |
| (j) $\frac{5}{2x+3} - 2x$ | $\frac{5-6x-4x^2}{2x+3}$ | | |
| (k) $\frac{1}{(x-1)^2} - \frac{2}{x-1}$ | $\frac{3-2x}{(x-1)^2}$ | | |

2. Write the following as a single fraction.

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|---|---------------------------|---|----------------------------------|
| (a) $\frac{1}{\frac{2}{x-4} - 3}$ | $\frac{x-4}{14-3x}$ | (g) $\frac{\frac{3}{x-2} + 2}{1 - \frac{4}{x}}$ | $\frac{x(2x+1)}{(x-2)(x-4)}$ |
| (b) $\frac{3}{\frac{1}{x-2} + \frac{2}{x}}$ | $\frac{3x(x-2)}{3x-4}$ | (h) $\frac{\frac{4}{3x}}{\frac{1}{x} - \frac{2}{x+1}}$ | $\frac{4(x+1)}{3(1-x)}$ |
| (c) $\frac{\frac{1}{x}}{2 - \frac{3}{x}}$ | $\frac{1}{2x-3}$ | (i) $\frac{\frac{1}{x-1} + \frac{3}{x-2}}{\frac{1}{x-1} - \frac{2}{x+3}}$ | $\frac{(4x-5)(x+3)}{(x-2)(5-x)}$ |
| (d) $\frac{\frac{4}{x-1}}{1 - \frac{3}{x-1}}$ | $\frac{4}{x-4}$ | (j) $\frac{1 - \frac{2}{x-3}}{\frac{1}{x+2} - 1}$ | $-\frac{(x-5)(x+2)}{(x-3)(x+1)}$ |
| (e) $\frac{2 + \frac{5}{x+3}}{\frac{1}{x+3} - 7}$ | $-\frac{2x+11}{7x+20}$ | (k) $\frac{\frac{a-b}{b} + \frac{c-b}{c}}{\frac{a}{b} - \frac{b}{c}}$ | □ |
| (f) $\frac{a - \frac{c}{x+b}}{e + \frac{f}{x+b}}$ | $\frac{ax+ab-c}{ex+eb+f}$ | (l) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$ | $\frac{3x+2}{2x+1}$ |

(m) $3 - \frac{\frac{5}{x-1}}{x - \frac{4}{x-1}}$. $\frac{3x^2 - 3x - 17}{x^2 - x - 4}$

(n) $\frac{1}{\frac{1}{x} + 1} - \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$. 0 (ho ho)

(o) $\frac{2}{3 + \frac{4}{5 + \frac{6}{x}}} - \frac{1}{2 + \frac{3}{4 + \frac{5}{x}}}$. $\frac{34x^2 + 65x + 30}{(19x + 18)(11x + 10)}$

3. Cancel the following to their simplest form.

(a) $\frac{x^2 + 3x}{2x + 6}$. □

(b) $\frac{x^2 - 1}{x^2 + 5x + 4}$. $\frac{x-1}{x+4}$

(c) $\frac{2x^2 + 7x - 4}{2x^2 + 9x - 5}$. $\frac{x+4}{x+5}$

(d) $\frac{4x^2 - 8x - 12}{2x^2 - 2x - 12}$. $\frac{2(x+1)}{(x+2)}$

(e) $\frac{4\pi x^2 - \pi}{12\pi x^2 - 10\pi x + 2\pi}$. $\frac{2x+1}{2(3x-1)}$

(f) $\frac{2a^2x^2 + a^2x - 10a^2}{2ax^2 - 2ax - 4a}$. $\frac{a(2x+5)}{2(x+1)}$

(g) $\frac{12x^4 + 17x^3 - 5x^2}{9x^3 - 25x}$. $\frac{x(4x-1)}{3x-5}$

(h) $\frac{x^2 + 6x + 8}{x^2 - 4x + 3} \div \frac{2x + 8}{x - 3}$. $\frac{x+2}{2(x-1)}$

(i) $\frac{\pi x^2 - \pi x - 2\pi}{x^3 + x^2 - 6x} \div \frac{2x + 2}{3x + 9}$. $\frac{3\pi}{2x}$

(j) $\frac{12x^2 + 10x + 2}{2x - 6} \div \frac{18x^2 + 15x + 3}{5x - 15}$. $\frac{5}{3}$

(k) $\frac{2x^2 + 2x - 4}{3x^3 - 3x} \div \frac{4x + 8}{9 + 9x}$. $\frac{3}{2x}$

(l) $\frac{4x^2 - 1}{2x^2 - 7x - 4} \div \frac{2x^2 + 5x - 3}{2x^2 - 2x - 24}$. 2

(m) $\frac{6\pi x^2 + 19\pi x + 10\pi}{3\pi x^3 + 4\pi x^2 - 7\pi x} \div \frac{6x + 15}{18x + 42}$. $\frac{2(3x+2)}{x(x-1)}$

4. Find the value of the capital letters which make the following identities correct.

(a) $\frac{A}{x} + \frac{B}{x+1} \equiv \frac{5x+2}{x(x+1)}$. A = 2, B = 3